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AREA COVERAGE BY LINE-OF-SIGHT RADIO

1. Problem Formulation:

We are concerned with line-of-sight coverage of an area where mobile terminals or fixed terminals are transmitting by radio from unspecified locations. The problem is to locate repeaters so that any such terminal will be in line-of-sight of repeaters and that there be reliable connections between every pair of terminals (and repeaters). More precisely we wish to minimize the installation cost and maintenance cost of the repeaters subject to a constraint on the reliability of service.

In general, determining if line-of-sight micro-wave transmission between two points is possible, involves taking into account many factors including wave-length (Fresnel zones), weather conditions (effective earth radius), antenna design, height, topography, etc.. Nevertheless, we shall assume that there are known functions $\ell(r,t)$ and $L(r_1, r_2)$ that are 1 if a repeater at location r can communicate with a terminal at location t and if a repeater at location r_1 can communicate with a repeater at location r_2 respectively and are 0 otherwise.

From purely topographical considerations it is ovious that the "flat terrain" problem and the "hilly terrain" problem should be handled seperately. For "flat terrain" the problem is homogeneous, i.e. installation costs, maintenance costs can be assumed equal at all locations and the transmission properties are identical at all points. The "flat terrain" problem is discussed in Section 3; a model is suggested for which we compute an optimal solution.

The primary concern in the ensuing paragraphs is with the "hilly terrain" problem. By that one should understand hilly topographies as well as flat topographies that cannot be viewed as homogeneous from a cost or transmission viewpoint.

In "hilly terrain" it is impractical to consider all possible locations of repeaters and terminals, which theoretically are infinite in number. We shall limit ourselves to a finite set R of possible repeaters locations and a finite set T of possible terminal locations. How the set R and T are chosen will be of great computational importance and will probably be chosen adaptively. But for the time being, we assume R and T known and fixed.

The principal and immediate interest is in an appropriate mathematical model of the situation and some indications on how to solve the problem. The first problem is the proper choice of reliability measure or grade of service. We assume that the radio network is for local distribution-collection of terminal traffic with rates small compared to the channel capacity so that throughput capacity is not a constraint. That is,if any path through the network exists for a given pair of terminals we assume there is sufficient capacity for traffic between them. Possible measure of network reliability that have proved useful in the analysis of communication network [Van Slyke and Frank; 1972] are the probability that all terminal pairs can communicate and the average fraction of terminal pairs which can communicate. However for network synthesis as destinguished from analysis these approaches appear too difficult both from computational and data collection points of view. This suggests the "deterministic" requirement that there exist k node disjoint paths between every terminal pair. This guarantees that at least k repeaters or line of sight links must fail before any terminal pair is disconnected. Let the cost of a repeater at location $r \in R$ be $c(r)$ and $c(R^o) = \{c(r) \mid r \in R^o\}$ where $R^o \subseteq R$. Then we can formulate:

Problem I: Find $R^* \subseteq R$ minimizing $c(R^*)$ subject to the constraint that for all $t \in T$ and $r \in R$ there exist k node disjoint paths from t to r in the network $N(t;R^*)$ where

$$N(t;R^*) = [R^* \cup \{t\}, \{(r_1,r_2) \mid L(r_1,r_2)=1\} \cup \{(r,t) \mid \ell(r,t)=1\}]$$

One might demand only that there be k node dispoint paths between every pair of terminals instead of between each terminal-repeater pair, but we are assumming that communication always takes place through a "station" which could be any of the repeaters. The analysis of the terminal to terminal model is similar in any case.

The following two propositions motivate a new Problem II,

Proposition 1: $|\{r \mid \ell(r,t)=1\} \cap R^*| \geq k$.

Proposition 2: For all $r \in R^*$, $|\{r_1 \mid L(r,r_1)=1\}| \geq k-1$.

If for each $r \in R$ there exists $t \in T$ such that $l(r,t)=0$ then $|\{r_1 \mid L(r,r_1)=1\}| \geq k$.

Problem II: Choose $R^* \subseteq R$ to minimize $C(R^*)$ subject to:

1. For all $t \in T$, $|\{r \mid \ell(r,t)=1\} \cap R^*| > k$
   (the k-fold set covering problem).
2. For all $r_1, r_2 \in R^*, r_1 \neq r_2$, there exist k node disjoint paths connecting r_1 to r_2.
   (the minimum cost redundant network problem).

The virtue of II - as compared to I is that II is an amalgam of two well studied network problems, the set covering problem [Garfinkel and Nemhauser; 1972] and a problem closely related to the minimum cost redundant network problem [Steiglitz, Weiner and Kleitman, 1969]. We can then attack the problem using previously developed techniques.

Problem I and II are related by:

Proposition 3: Any R^* satisfying the constraints of II also satisfies the constraints of I.

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Proof: Suppose R* satisfies the constraints of Problem 2 and violates those of Problem 1. Then there exists a terminal t_0 and a repeater r such that there are not k node disjoint paths connecting them in N (t_0; R*). According to the Menger Graph Theorem [Hu; 1969, p.122] there are k-1 repeaters $r_1,\ldots,r_{k-1}$ not including r_0 such that when these repeaters are removed from R* there is no path from t to r_0. But this leads to a contradiction. Removing k-1 repeaters cannot disconnect t_0 from the net by 1 of Problem 2. Thus there is an arc (t_0,r') with $\ell(r',t^0)=1$. Similarly k-1 repeater cannot disconnect r' from r_0 by 2. Thus there is a path from t_0 to r.

The converse is not necessarily true; that is, there may be feasible solutions to I which are not feasible to II. Figure 1 depicts a counter-example to a possible converse for k=2.

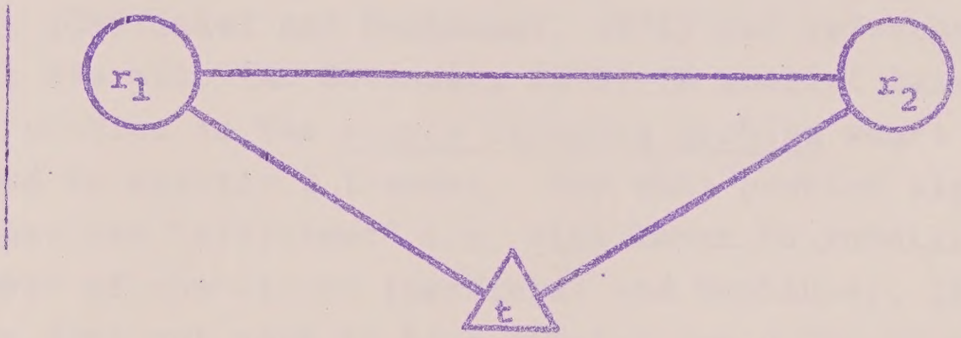


Figure 1.

However, by Proposition 3 solving II is fail-safe in the sense that the solution obtained will always be feasible for Problem 1.

Feasible solutions to I which are not feasible in II will be quite rare. In order for this to happen, there would have to be two repeaters r_1 and r_2 in R* (where R* is an assumed solution to I) for which there are not k node disjoint paths joining them yet for which all terminals in the vicinity of r_1 can communicate by k node disjoint paths with r_2 and conversely. This doesn't happen unless there are very few terminals and many repeaters near r_1 and r_2 which from the physical nature of the problem is highly unlikely. On the other hand, as pointed out above, artificial counter-examples to a possible converse of Proposition 3 can easily be constructed.

2. Computational Techniques:

The following problem will be referred to as a k-cover problem. Let N be a bipartite graph with edges E and nodes (R,T). The edges E connect nodes of R to nodes of T. Then the following is a k-cover problem:

Find $R^* \subset R$ such that valence of each T-node in N^* is at least k and such that the cardinality of R^* is minimum (number of elements of R^*), where N^* is the subgraph obtained by deleting all nodes $R \setminus R^*$ (where $\setminus$ indicates set difference) and all edges connected to these nodes. This is also known as the α-width problem [Ford and Fulkerson, 1962], [Ryser,1963]

The 1-Cover Problem is the classical set covering problem. Extensive research has been and is being done on the 1-cover problem, see e.g. [Garfinkel and Nemhauser, 1972] and references mentioned therein, see also [M. Balinski, 1970] (A special case of the 1-cover problem is the simple covering problem where each R-node is connected to exactly 2 T-nodes. For this problem algorithms are known that are "efficient" i.e. with known polynomial bounds on the number of operations [Garfinkel and Nemhauser, 1972] So far, practice does not seem to have singled out a "best" algorithm to solve the general 1-cover problem . But in any case those available seem to be much more efficient than solving the problem as a straight integer programming problem.

a. Every k-cover problem is equivalent to a 1-cover problem.

To establish the assertion let us consider the case when k=2. The generalization to arbitrary k is important but it is easier to see the proof for the case k=2.

Let us consider a bipartite graph constructed in the following manner: Start with a 2-covering problem. Let L be the number of elements in R. Take L copies of the T-nodes. Connect node r_1 to all copies of node t_j (if r_i, t_j) ϵ E of the 2-cover problem, except for the copy of t_j in the i^{th} copy of T. (See Figure 2)

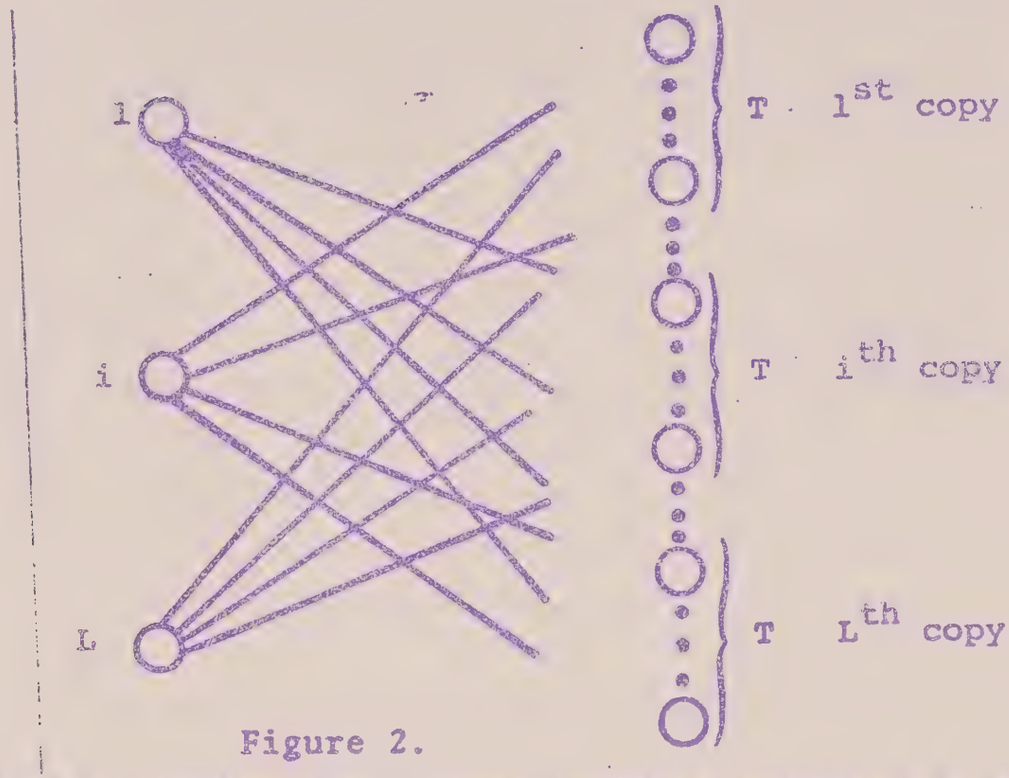


Figure 2.

We call this problem the 1-cover problem <u>generated</u> by the 2-cover problem.

<u>Proposition</u>: Every solution to the 2-cover problem yields a solution to the generated 1-cover problem and conversely.

In particular, the optimal solution of the 2-cover problem yields an optimal solution to the generated 1-cover problem and conversely.

<u>Proof</u> The first part (a 2-cover solution yields a solution to the generated 1-cover) is trivial. In the other

direction, suppose the n^{th} copy of t_j is "covered" by node r_i the r_is optimal 1-cover of the generated 1-cover problem, but in the i^{th} copy, t_j must be "covered"by another node in R. So every node is "covered" by at least 2-elements of R. The optimality statements follows readily.

For k-covers (k>2) a similar proposition can be found but then the number of copies of T needed (L in these case) go up exponentially.(We suspect that this type of transformation is similar to reducing general integer variables in integer programs to 0-1 variables.)

Important remarks: If the problem is formulated as a k-cover problem, one should realize that by the above results one might suspect that the k-cover problem is much more complicated than the I-cover problem. This is corrobarated by the experience with integer programming algorithms. The only class of problems which can be solved with any form of computational success are those involving only 0 and 1's.

b. The Covering Problem as an Integer Program. Let A be a (T,R) matrix (where the rows correspond to terminals and the columns to repeaters). The size of A is $|T|x|R|$. We have that a_{ij}= 1 or 0 depending on whether terminal t_i is "visible" or not to repeater r_j. In terms of the bipartite graph N, the entry a_{ij}=1 if there is an edge between nodes r_i and t_j, the entry a_{ij}=0 otherwise. We can then formulate the k-cover problem as follows:

$$\text{(I.P)} \quad \begin{aligned} &\text{Min} \ \Sigma x_j \\ &\text{such that} \ \sum_j a_{ij}x_j \geq k \quad i=1,\ldots,|T| \\ &\text{with} \quad x_j=1 \text{ or } 0 \end{aligned}$$

This is an integer program (sometimes called a program in boolean variables).A number of algorithms to solve integer

programs are known. See: [Garfinkel and Nemhauser, 1972] for a survey. These algorithm offer little hope in solving the location problem arising in the Packet Radio project in that |R| and|T|, i.e. the number of variables and the number of constraints respectively, are much too large for existing methods for k-cover problems.

c. Approximate to the k-cover Method by Linear Programming:

The integer programming problem formulated above can be replaced by a linear program

$$\text{Minimize } \sum_j x_j$$

$$\text{(L.P.) such that } \sum_j a_{ij}x_j \geq k \quad i=1,\ldots,|T|.$$

$$0 \leq x_j \leq 1$$

Where the constraint $x_j=0$ or 1 has been replaced by the constraint $0 \leq x_j \leq 1$. The difference is obvious, the solution to (L.P.) will contain fractional values, but we note that if a variable is "profitable", it will usually be made as large as possible. The constraints $\sum_j a_{ij} x_j \geq k$ do not generate upper bounds on the x_j's. Thus one might expect many 0 and 1's in the optimal solution of the L.P. problem.

An upper bound for the optimal solution to the (I.P.) integer program can be obtained by "pushing" all the fractional x_j's appearing in the optimal solution to L. Obviously, that yields an upper bound on Min $\sum x_j$. The optimal solution of (L.P) yields a lower boun

There are many ways to improve the above solution. (without actually going all the way to integer programming). Let us suggest two.

Scheme 1. Let x^{opt} be the optimal solution of the (L.P.) problem. Set $x^*_j=1$ if $x^{opt}=1$ and $x^*_j=0$ otherwise.
Let $k^*_i=\text{Max } \{0, k-\sum a_{ij}x^*_j\}$. Construct A* as follows: Remove from A all columns j such that $x^*_j=1$ and remove all rows with $k^*_i=0$. Set $k^*=\{k^*_i \mid k^*_i \neq 0\}$. You have now a reduced problem. It

is again an integer program.

$$\text{Min} \Sigma x_j$$

$$\text{(I.P.S.1)} \quad \text{s.t} \quad A^* x \geq k^*$$

$$x_j = 0 \text{ or } 1$$

The problem is substantially reduced in size. We can now use an integer programming algorithm.

Scheme 2. Once the solution to the L.P. is obtained. A branch-and-bound algorithm can be developed to be continued until a sufficiently small difference exists between the best solution and the smallest upper bound obtained so far. Note that by the remarks preceeding Scheme 1 it is always very easy to obtain upper and lower bounds.

There is also the possibility to use "integer cuts", see [Garfinkel and Nemhauser, 1972]. This is integer programming. However, there is a possibility that for k-cover problem good cuts can actually be constructed and recognized. That is one direction of research (that might become imperative) but which has not been pursued. Success in characterizing cuts has been achieved in the past for highly structured problems, see [J.Edmonds, 1962] and [R.Fulkerson, 1971].

d. A Network Flow Problem with Concave Cost. For the sake of completeness we record one other formulation. The problem is to find a feasible flow (in the network to be described below) at minimum cost. One possible advantage is the potential use of algorithms specifically developed for fixed charge problems or branch-and-bounds methods on nets, see e.g. [Martin, 1974,] and [Walkup, 1967]. We consider the bipartite graph N as described in the beginning of this section (2), Figure 3.

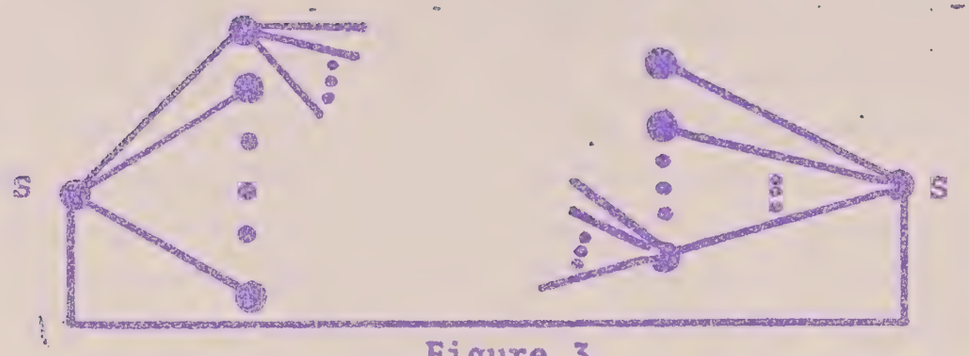


Figure 3.

to which we add a source s and a sink S. Let f (x,y) be the flow between nodes x and y, we have the following constraints:

$f(x,y) \geq 0$ for all x,y

$f(s,y) \leq c(s,y)$ for all $y \in R$

$f(x,S) \geq p(x,S)$ for all $x \in T$

and $f(x,y)$ integer

The upper bound c (s,y) is set equal to the valence of $y \in R$ minus 1 and for all $x \in T$ $p(x,S) = k$. The cost $a[f(x,y)]=0$ for all arcs except for arcs (s,y), $y \in R$ where.

$a[f(s,y)] = 0$ if $f(s,y)=0$

$a[f(s,y)] = 1$ otherwise.

e. A Heuristic Approach to the k-cover Problem. Given the limited success of integer programming algorithms in solving large scale problems, we have been led to consider heuristic methods to find good solutions to the k-cover (of terminals by repeaters) problem which is typically large scale. It is intuitively appealing to consider a terminal as particularly critical if it is adjacent to few repeaters. (In the extreme cases, if a terminal has fewer than k adjacent repeaters, the problem is infeasible and if it has exactly k adjacent repeaters, all of them must be chosen for any feasible solution.) Similarly, a repeater is desirable if it is adjacent to a large number of terminals, especially if the terminals are highly critical. The heuristic algorithms described below systemizes these intuitive notions in the search of a "good" solution.

Again, consider the problem in matrix form, but this time we imbed the problem in a more general class when we can require a different cover multiple for each terminal. The more general problem is:

$$\text{(I.P.H)} \quad \text{Min } \Sigma_j x_j,$$

$$\text{such that } \Sigma_j \, a_{ij} \, x_j \geq k_i, \quad i = 1, \ldots, |T|,$$

$$\text{with } x_j = 0 \text{ or } 1,$$

where k_i represents the cover multiple required for terminal i. If $k_i \leq 0$, then no repeater is needed to cover terminal i.

One iteration of the heuristic method consists in passing from a problem of the type (I.P.H.) to an "equivalent" problem by fixing one of the variables, say x_s, as its upper bound 1, and putting the index s in an index set J. This implies the selection of the corresponding repeater. The new problem, denoted by $(\text{I.P.H.})_n$ is obtained

There obviously exists a function σ - a selection criterion - which will guarantee the equivalence of (I.P.H.) and (I.P.H.) at each iteration and consequently, the resulting heuristic solution of the original problem. However, applying this funciton σ to [A,k] might involve no less than solving (I.P.H.). We use a heuristic motivated by the considerations mentioned at the beginning of this section. The adjusted A and k are used to compute the "probability" that a given x_j belongs to the optimal solution, by this we mean that to each column of the adjusted matrix A we associated a nonnegative number ω_j such that;

$$P_j = \omega_j (\Sigma\ \omega_j |\ j \notin J)^{-1}$$

represents very loosely speaking - the probability that x_j belongs to the optimal solution. We select x_s, $s \notin J$ if $P_s \geq P_j$ for $j \notin J$ or equivalently if $\omega_s \geq \omega_j$ for $j \notin J$. The selection criterion σ will be completely determined if we specify a method to compute the ω_j.

In the selection of these weights ω_j, we must take into consideration the effort involved in the computation as well as the reliability of the resultant selection. We have used four such weights:

We first define;

$$k_i^* = 1/2\ [|k_i| + k_i]$$

Observe that $k_i^* = 0$ if $k_i \leq 0$ and $k_i^* = k_i$ otherwise.

We set;

$$\omega_j^{(1)}\ (A,k) = \Sigma_i\ \left(\frac{k_i^*}{\Sigma_j\ a_{ij} - k_i^*}\right) a_{ij} \qquad j \notin J$$

$$\omega_j^{(2)}(A,k) = \Sigma_i \left(\frac{k_i^*}{\Sigma_j a_{ij}}\right) a_{ij} \qquad j \notin J$$

$$\omega_j^{(3)}(A,k) = \Sigma_i \left(\frac{1}{\Sigma_j a_{ij} - k_i^*}\right) a_{ij} \qquad j \notin J$$

$$\omega_j^{(4)}(A,k) = \Sigma_i \left(\frac{1}{\Sigma_j a_{ij}}\right) a_{ij} \qquad j \notin J$$

The entries between parentheses () in the definition of weights ω_j, measures how critical terminal i is. If $k_i^* = 0$, the terminal does not need covering then $k_i^* / (\Sigma_{j \in J} a_{ij} - k_i^*) = 0$ (for our purposes $0/\infty = 0$). On the other hand, if $\Sigma_j a_{ij} = k_i$, namely there are exactly enough repeaters to cover the terminal then the weight is infinite. A repeater will be preferred to another one if it covers more critical terminals.

3. The "Flat Terrain" Model. A Proposed Solution

a. Problem Formulation.

It is assumed that:

(i) the terrain is nearly flat

(ii) The installation (and maintenance) expenses for repeaters are independent of location.

(iii) transmission characteristics are invariant with location. A repeater communicates with a terminal if and only if they are less than a fixed distance d_t apart. The maximum distance for communication between repeaters is assumed to be d_r (In practice d_r is substantially larger than d_t because repeater antennas are higher than terminal antennas). The area coverage by L.O.S. radio can again be separated into two parts, a covering problem and a connectedness problem (see Section I Problem II). Let P be the 2-dimensional plane.

Covering Problem. Find a minimal covering of P by discs of radius d_t such that every point of P is covered a least k times.

Connectedness problem. Let G= (N,E) be the graph obtained as follows: The nodes N are the centers of the discs used in the minimal covering of P. The edges E of G are obtained by connecting two nodes if their distance is less or equal to d_r. The graph G is to be q-connected (reliability).

Since we are considering an infinite plane one can no longer define minimality of a cover in terms of its cardinality. There are various proceedures to define minimality, for example, the cover with the smallest percentage of area wasted or if $\lim_{r\to\infty} \frac{1}{r^2} \delta_C(r)$ is minimized over the space of all covers C of P that

satisfy the covering and connectedness constraints, where $\delta_C(r)$ is the number of discs of C whose interior intersect a circle of radius r centered at the origin (an arbitary but fixed point of P).

b. Solutions, A conjecture: An optimal solution to the above problem is known for k=1, $q \leq 6$ and $d_r/d_t \geq \sqrt{3}$ (that is the maximum distance for transmission between repeaters is at least 73.3% larger than that between terminals and repeaters). The problem is then the standard covering of the plane by discs of fixed radius and with least overlap. In [F. Toth, 1953] it is shown that the optimal solution is given by arrangement found below, which consists in placing a circle of radius d_t at each vertex of a regular triangular tessalation whose grid points (vertices) are $d_t\sqrt{3}$ apart. Since $d_r \geq d_t\sqrt{3}$ it follows that the resulting graph G contains as subgraph the regular triangular tesselation whose grid points (vertices) are $d_t\sqrt{3}$ apart which is 6-connected. In Figure 4 the area of the discs covered twice is shaded.

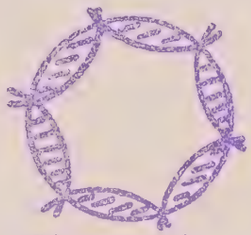
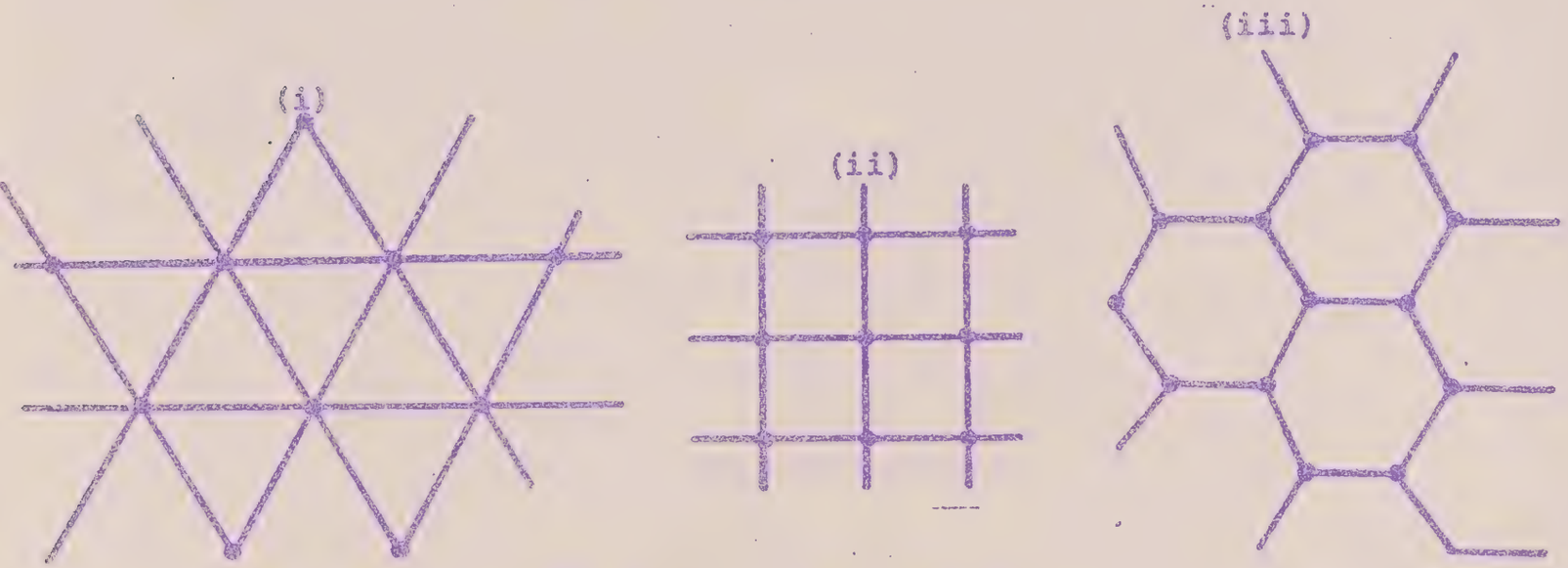


Figure 4

We do not consider any other cases for k=1 since $q \leq 6$ and $d_r \geq d_t\sqrt{3}$ will always be satisfied in practice.

For k=2, the optimal solution is not known (whatever be q and d_r/d_t). However, due to the inherent symmetry of the problem, we are ready to conjecture that the centers of the optimal solution produces a regular grid of points in P. Making this conjecture our working hypothesis, there are only three cases to consider, the three.regular tesselations: (i) the tesselation of P by equilateral triangles, (ii) by squares, (iii) by hexagons. [Coxeter, 1961]



The Regular Tesselations

Figure 5

Each vertex of the tesselation corresponds to a repeater and if every point in P is to covered twice then the distance between two adjacent repeaters should not exceed d_t. Assuming that d_t is the distance between two adjacent vertices of the tesselations, then in the triangular tesselation one finds 6 repeaters at distance d_t, 6 repeaters at distance $\sqrt{3}d_t$, 6 repeaters at distance $2d_t$ ...from any given repeater. In the square tesselation one finds 4 repeaters at distance d_t, 4 repeaters at distance $\sqrt{2}d_t$, 4 repeaters at distance $2d_t$...from any given repeater.

Finally for the hexagonal tesselation there are 3 repeaters at distance d_t, 6 repeaters at distance $\sqrt{3}\, d_t$, 3 repeaters at distance $2\, d_t$, . . . from any repeater. With each tesselation one can associate a repeater density. It is easy to see that there is 1 node:

per $\sqrt{3}/2\ d_t^2$ units of area in case (i)

per d_t^2 units of area in case (ii)

per $\frac{3\sqrt{3}}{4}\ d_t^2$ units of area in case (iii)

Assuming that $\frac{3\sqrt{3}}{4}\ d_t^2$ is 1 unit of area it follows that the repeater density is

1.5 for triangular tesselations

1.293 for square tesselations

and 1 for hexagonal tesselations

In other words, design (i) requires 50% more equipment than (iii) and (ii) require ~30% more equipment than (iii). Obviously, if a regular grid yields the optimal solution then the one created by the hexagonal tesselation (iii) is the optimal solution. If $q \leq 3$ and $d_t \leq d_r$ then the corresponding graph G contains the subgraph given by the tesselation (iii) which is obviously 3-connected. If $d_r \geq \sqrt{3}\ d_t$ and $q \leq 6$ again the connectedness constraint is satisfied.

One should observe some inefficiencies in this covering. In particular some areas are covered by three separate repeaters rather than 2 (as would ideally be the case). However, if we define the thickness of a cover as being the average

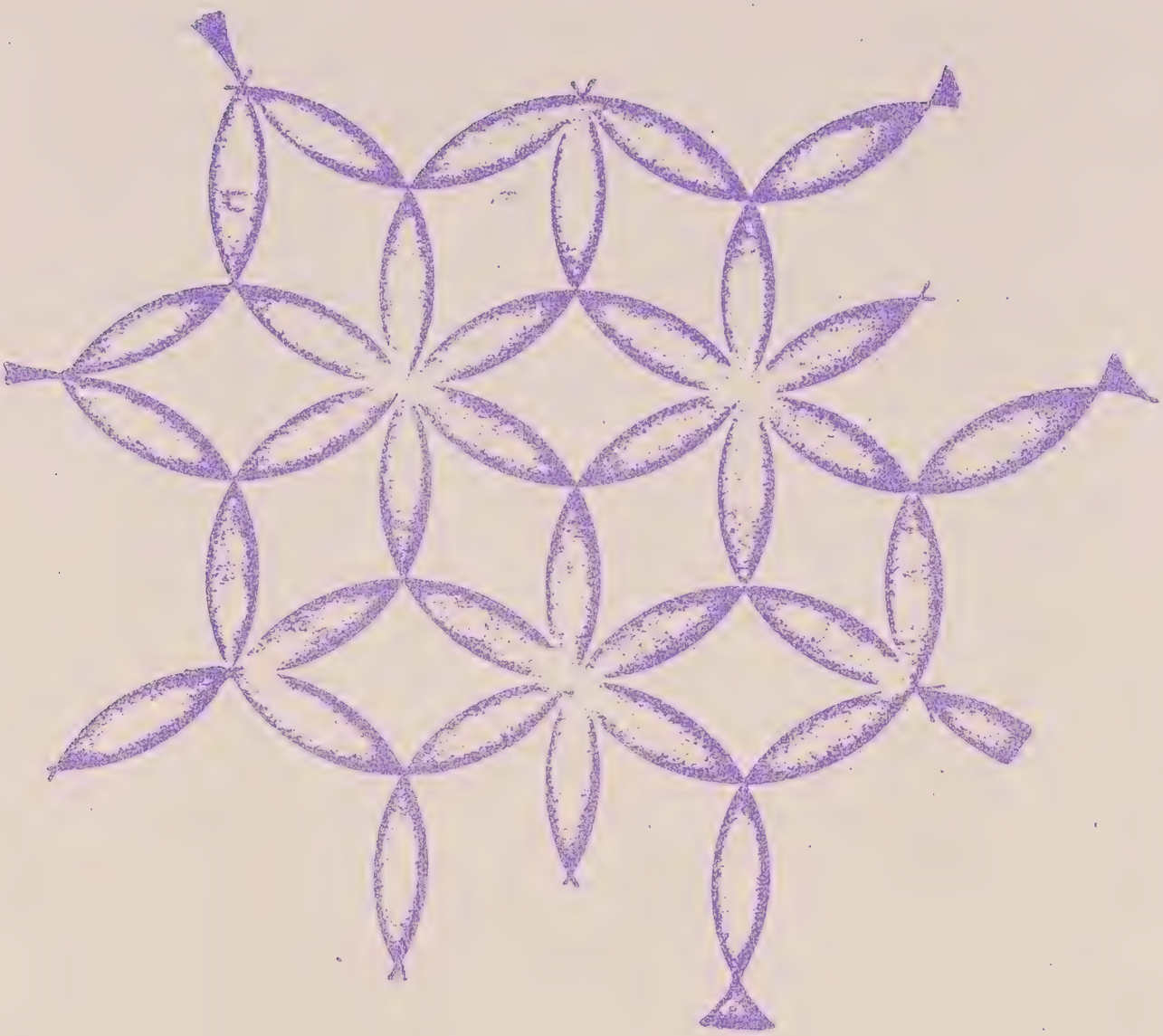


Figure 6.

"thickness" of the layer of discs covering the plane then the optimal solution for k=1 has thickness 1.209 whereas the thickness of the conjectured optimal solution for k=2 is 2.418. Thus, in both cases, we have a 21% "waste." In Figure 6 a solution is shown with the area covered three times shaded. The question for k>2 is open.

Remark : The "flat terrain" results yield obvious lower bounds for the "hilly terrain" problem.

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